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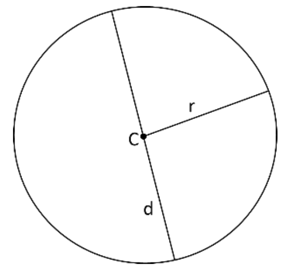
Arcs and Sectors Practice Worksheet

Circles

In order to understand arc length and sector area, we have to first know a few things about circles.

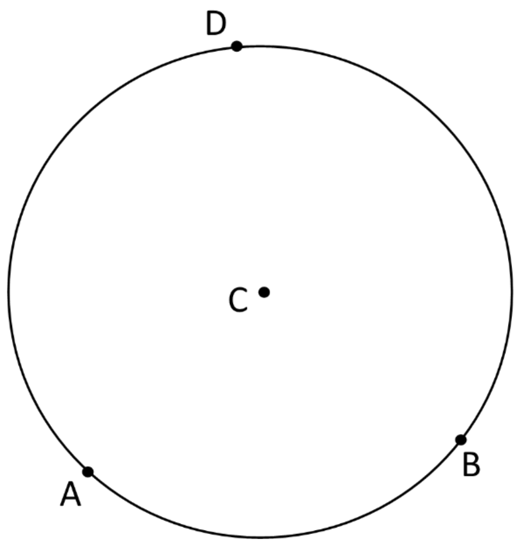
In this circle, we can see the radius labeled as r and the diameter labeled as d.



A circle is usually named for the center point, so in this case we would call it circle C.

Now I want you to take the circle below and draw a line from point C to A and a line from C to B.

What you have just drawn is called a **central angle**. The central angle in this case is $\angle ACB$ or $\angle BCA$. The portion of the circumference (or edge) of the circle between A and B is called arc AB, or $\widehat{AB}$. We measure the arc by its central angle measurement. An arc with a central angle of 43° would be said to have a measure of 43°.



Arcs and Sectors

Arcs look like slices of pizza. The arc length is the amount of crust that particular pizza has. The area of the sector is the amount of pizza in the whole slice. We can find either one of these by knowing the central angle. For example, let's say that in a certain circle the measure of arc **PZ** is 90° and the radius is 7 inches. A 90° angle is one fourth of the whole circle, since a whole circle is 360°. If we can find the circumference and area of the whole circle, then we can find the portion of the circle we have by multiplying that by ¼.

The area of a circle is found through the formula $A = \pi r^2$ and the circumference is found by $C = 2\pi r$. For the circle below, this means that

$$Arc\ length = \frac{1}{4} \times Circumference = \frac{1}{4} 2\pi r = \frac{1}{4} 2\pi(7) = \frac{14\pi}{4} \approx 11in$$

$$Sector\ Area = \frac{1}{4} \times Area = \frac{1}{4} \pi r^2 = \frac{1}{4} \pi(7)^2 = \frac{49\pi}{4} \approx 38.48in^2$$

Note that the arc length is always given in the linear units of the radius and diameter and sector area is given in square units, because it is a square measurement.

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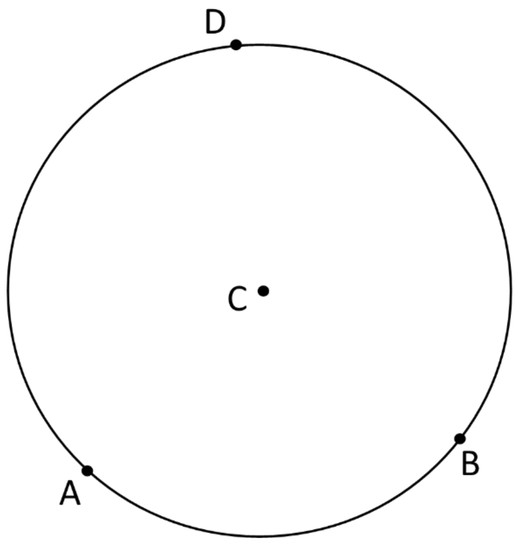
If we have an angle that can't be easily computed into a fraction of a circle, we can find the fraction by taking the measure of the angle over 360° and simplifying the resulting fraction.

Now let's go back to the circle you were drawing on before and draw a line from A to D

and a line from D to B.



This is called an **inscribed angle**. We say that $\angle ADB$ *or* $\angle BDA$ is the angle inscribed in $\widehat{AB}$, or that $\widehat{AB}$ **subtends** $\angle ADB$. The measure of an inscribed angle is half the measure of the central angle subtending the same arc. In this case that would mean that $m\angle ADB = \frac{1}{2} m\angle ACB$. The central angle will always have a measure twice that of the inscribed angle, provided that subtend the same arc.

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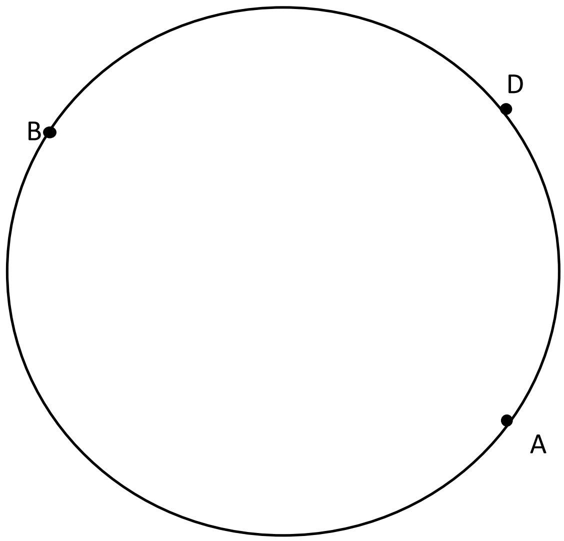
It is important to note a special case here for semicircles. If the two points on the circumference of the circle make a diameter of the circle (that is to say, the line between them goes through the center of the circle), then we know the central angle they create is 180° since it creates a semicircle. If the central angle is 180°, then the measure of the inscribed angle must be 90°, since the inscribed angle is half the measure of the central angle.

For the circle below, let $\overline{AB}$ be a diameter of circle C. If we draw lines AD and BD, what must be true of $\angle ADB$? Since $\angle ACB = 180°$, we know that $\angle ADB = 90°$. What do we now know about triangle ΔADB? Try drawing an inscribed angle subtending $\widehat{AB}$ that is not a right angle.



It's impossible!

Practice Problems

1. Find the arc length and sector area of an arc with a central angle of 45° with a radius of 2 miles.
2. If the arc length of a circle with a diameter of 10π is 2π, what is the measure of the arc?