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Complex Numbers Practice Worksheet

Complex numbers are numbers with a real portion and an imaginary portion. They are written as $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. In order to understand the workings of complex numbers, we first have to understand $i$.

Because $i = \sqrt{-1}$, it has some interesting properties. For example, we can see that $i^2 = \sqrt{-1}^2 = -1$, since the square root and the square are inverse operations. Taking this a few more steps, we can generate the table:

| | |
|---|---|
| $i$ | $i$ |
| $i^2$ | $-1$ |
| $i^3$ | $-i$ |
| $i^4$ | $1$ |

If we continue to increase the power of $i$, we just cycle through these values. This lets us simplify expressions like $i^{27}$. We know that $i^4 = -1$, and $i^{27} = i^{24} \cdot i^3 = 1 \cdot i^3 = -i$.

Practice: Use the properties of complex numbers to simplify the following expressions.

1. $i^{12}$

2. $i^{50}$

3. $-4i^2$

Every complex number has a twin called its **complex conjugate**. The number $a + bi$ has a complex conjugate $a - bi$. These numbers are not identical twins, since they obviously don't look exactly alike, but they occur at the same time as the solutions of a polynomial with real coefficients. As an example let's look at the polynomial $f(x) = x^3 + 5x^2 + 9x + 5$. We know that one of the roots of this function is -1. When we try to find the other two roots, since we know there has to be three of them counting the complex numbers, we discover that the other roots are complex. One of the roots is $-2 - i$, and we want to find the other complex root. Since they occur in pairs of complex conjugates, we know it must be $-2 + i$.

Practice: Write the complex conjugate of each complex number.

4. $5 + 2i$

5. $-4 + i$

6. $i - 7$

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Besides finding roots of a function, complex conjugates also allow us to simplify expressions and rationalize denominators. Since $i = \sqrt{-1}$, any denominator that has a complex number is in is not rationalized. We use a special property of complex conjugates to simplify such fractions.
This property tells us that the product of a complex number and its conjugate is always a real number. Let's see this in practice. Keep in mind the rules of FOIL while doing this!

$$(2+i)(2-i) = 4 + 2i - 2i - i^2 = 4 - (-1) = 4 + 1 = 5$$

Whenever we are finding the product of two complex numbers we know that the middle two terms are going to cancel (the O and I in our FOIL). The last term is going to have an $i^2$ in it, and we know that $i^2 = \sqrt{-1}^2 = -1$, therefore it will become a real number.

We use this to rationalize the denominators of fractions as follows:

$$\frac{3-i}{5+2i} = \frac{3-i}{5+2i} \cdot \frac{5-2i}{5-2i} = \frac{15 - 5i - 6i + 2i^2}{25 + 10i - 10i - 4i^2} = \frac{15 - 11i - 2}{25 + 4} = \frac{13 - 11i}{29}$$

Now you try it!

Practice: Rationalize the denominator of each of the complex fractions.

7. $\frac{3+i}{5+i}$

8. $\frac{3+2i}{i-6}$

9. $\frac{6}{7+2i}$